

# QCD SUM RULES STUDY OF THE $J^{PC} = 1^{--}$ CHARMONIUM $Y$ MESONS

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We use QCD sum rules to test the nature of the recently observed mesons  $Y(4260)$ ,  $Y(4350)$  and  $Y(4660)$ , assumed to be exotic four-quark ( $c\bar{c}q\bar{q}$ ) or ( $c\bar{c}s\bar{s}$ ) states with  $J^{PC} = 1^{--}$ . We work at leading order in  $\alpha_s$ , consider the contributions of higher dimension condensates and keep terms which are linear in the strange quark mass  $m_s$ . We find for the ( $c\bar{c}s\bar{s}$ ) state a mass  $m_Y = (4.65 \pm 0.10)$  GeV which is compatible with the experimental candidate  $Y(4660)$ , while for the ( $c\bar{c}q\bar{q}$ ) state we find a mass  $m_Y = (4.49 \pm 0.11)$  GeV, which is higher than the mass of the experimental candidate  $Y(4350)$ . With the tetraquark structure we are working we can not explain the  $Y(4260)$  as a tetraquark state. We also consider molecular  $D_{s0}\bar{D}_s^*$  and  $D_0\bar{D}^*$  states. For the  $D_{s0}\bar{D}_s^*$  molecular state we get  $m_{D_{s0}\bar{D}_s^*} = (4.42 \pm 0.10)$  GeV which is consistent, considering the errors, with the mass of the meson  $Y(4350)$  and for the  $D_0\bar{D}^*$  molecular state we get  $m_{D_0\bar{D}^*} = (4.27 \pm 0.10)$  GeV in excellent agreement with the mass of the meson  $Y(4260)$ .

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## I. INTRODUCTION

Recent experimental observations of new particles at  $B$ -factories, have revitalized the field of hadron spectroscopy. While some of these observed states, like the  $X(3940)$  [1], can be understood as  $c\bar{c}$  states [2], these observations suggest that the spectrum of the charmonium states is much more rich than suggested by the quark-antiquark model and may include tetraquark or molecular states. In particular, the  $X(3872)$  [3], with quantum numbers  $J^{PC} = 1^{++}$ , was the first observed state that did not fit easily the charmonium spectrum. It also presents a strong isospin violating decay that disfavors a  $c\bar{c}$  assignment. The  $Z^+(4430)$ , recently observed in the  $Z^+ \rightarrow \psi'\pi^+$  decay mode [4], is the most intriguing one since, being a charged state, it can not be a pure  $c\bar{c}$  state.

Besides the  $X(3872)$  and  $Z^+(4430)$  mesons, at least three peaks of the  $J^{PC} = 1^{--}$  family: the  $Y(4260)$  [5], the  $Y(4325)$  [6] or/and  $Y(4360)$  [7] (that here we call  $Y(4350)$ ) and the  $Y(4660)$  [7] can not be easily fitted in the charmonium spectrum [8]. Since the only observed decay channels for these states are those containing the  $J/\psi$  (for  $Y(4260)$ ) or  $\psi'$  (for the others) plus a pair of pions, in ref. [2] this was considered as an indication that these mesons contain a particular charmonium resonance,  $J/\psi$  or  $\psi'$ , that stays intact inside a more complex hadronic structure.

A critical information for understanding the structure of these states is whether the pion pair comes from a resonance state. From the di-pion invariant mass spectra shown in ref. [9] there is some indication that only the  $Y(4660)$  has a well defined intermediate state consistent with  $f_0(980)$  [9]. Due to this fact and the proximity of the mass of the  $\psi' - f_0(980)$  system with the mass of the  $Y(4660)$  state, in ref. [10], the  $Y(4660)$  was considered as a  $f_0(980) \psi'$  bound state. The  $Y(4660)$  was also suggested to be a baryonium state [11] and a canonical  $5^3S_1 c\bar{c}$  state [12].

In the case of  $Y(4260)$ , in ref. [13] it was considered as a  $sc$ -scalar-diquark  $\bar{s}\bar{c}$ -scalar-antidiquark in a  $P$ -wave state. In a naive estimate, the mass of a  $sc$ -scalar-diquark would be approximately equal to the mass of the  $D_s$  meson. Since a unit of angular momentum can be estimated as the mass difference between the nucleon and the lowest-lying odd parity excited state, and it is about 600 MeV [14], one would expect the mass of a  $sc$ -scalar-diquark  $\bar{s}\bar{c}$ -scalar-antidiquark in a  $P$ -wave state about 4540 MeV, almost 300 MeV above the  $Y(4260)$  mass.

Here we use the QCD sum rules (QCDSR) [15, 16, 17] to study the two-point function for a diquark-antidiquark tetraquark state with a symmetric spin distribution:  $[cs]_{S=1}[\bar{c}\bar{s}]_{S=0} + [cs]_{S=0}[\bar{c}\bar{s}]_{S=1}$ , to see if any of these new  $Y$  mesons can be described by such a current. In previous calculations, the QCDSR approach was used to study the  $X(3872)$  considered as a diquark-antidiquark state [18], and the

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$Z^+(4430)$  meson, considered as a  $D^*D_1$  molecular state [19]. In both cases a very good agreement with the experimental mass was obtained.

## II. THE TWO-POINT CORRELATOR

The lowest-dimension interpolating operator for describing a  $J^{PC} = 1^{--}$  state with the symmetric spin distribution:  $[cs]_{S=0}[\bar{c}\bar{s}]_{S=1} + [cs]_{S=1}[\bar{c}\bar{s}]_{S=0}$  is given by:

$$j_\mu = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(s_a^T C \gamma_5 c_b)(\bar{s}_d \gamma_\mu \gamma_5 C \bar{c}_e^T) + (s_a^T C \gamma_5 \gamma_\mu c_b)(\bar{s}_d \gamma_5 C \bar{c}_e^T)] , \quad (1)$$

where  $a, b, c, \dots$  are color indices and  $C$  is the charge conjugation matrix.

The two-point correlation function is given by:

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle = -\Pi(q^2)(g_{\mu\nu}q^2 - q_\mu q_\nu), \quad (2)$$

from where we get

$$\Pi_\mu^\mu(q) = -3q^2\Pi(q^2), \quad (3)$$

The calculation of the phenomenological side at the hadron level proceeds by writing a dispersion relation for the invariant function in Eq. (3):

$$\Pi^{phen}(q^2) = - \int ds \frac{\rho(s)}{q^2 - s + i\epsilon} + \dots, \quad (4)$$

where  $\rho$  is the spectral density and the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

$$\rho(s) = \lambda^2 \delta(s - m_Y^2) + \rho^{cont}(s), \quad (5)$$

where  $\lambda$  is proportional to the meson decay constant  $f_Y$ , which parametrizes the coupling of the vector meson,  $Y$ , to the current  $j_\mu$ :

$$\langle 0 | j_\mu | Y \rangle = f_Y m_Y^4 \epsilon_\mu = \lambda m_Y \epsilon_\mu. \quad (6)$$

For simplicity, it is assumed that the continuum contribution to the spectral density,  $\rho^{cont}(s)$  in Eq. (5), vanishes below a certain continuum threshold  $s_0$ . Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz [20]

$$\rho^{cont}(s) = \rho^{OPE}(s) \Theta(s - s_0), \quad (7)$$

with

$$\rho^{OPE}(s) = \frac{1}{\pi} \text{Im}[\Pi^{OPE}(s)]. \quad (8)$$

In the OPE side, we work at leading order in  $\alpha_s$  and consider the contributions of condensates up to dimension six. We keep the term which is linear in the strange-quark mass  $m_s$ . We use the momentum space expression for the charm quark propagator, while the light-quark part of the correlation function is calculated in the coordinate-space. The correlation function,  $\Pi(q^2)$ , in the OPE side can also be written in terms of a dispersion relation:

$$\Pi^{OPE}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}. \quad (9)$$

Therefore, after making an inverse-Laplace (or Borel) transform on both sides, and transferring the continuum contribution to the OPE side, the sum rule for the vector meson  $Y$  can be written as

$$\lambda^2 e^{-m_Y^2/M^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M^2} \rho(s), \quad (10)$$

where

$$\rho^{OPE}(s) = \rho^{pert}(s) + \rho^{\langle \bar{s}s \rangle}(s) + \rho^{\langle G^2 \rangle}(s) + \rho^{mix}(s) + \rho^{\langle \bar{s}s \rangle^2}(s), \quad (11)$$

with

$$\begin{aligned} \rho^{pert}(s) &= -\frac{1}{2^8 3 \pi^6 s} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s]^3 [m_c^2 - 2m_c^2(\alpha+\beta) + \alpha\beta s], \\ \rho^{\langle \bar{s}s \rangle}(s) &= -\frac{3m_s m_c^2 \langle \bar{s}s \rangle}{2^4 \pi^4 s} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} [(\alpha+\beta)m_c^2 - \alpha\beta s], \\ \rho^{\langle G^2 \rangle}(s) &= \frac{m_c^2 \langle g^2 G^2 \rangle}{3^2 2^9 \pi^6 s} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) \left[ 4(2\alpha+2\beta-1)m_c^2 - \frac{3m_c^2\beta}{\alpha} - \beta s(7\alpha-3) \right], \\ \rho_1^{mix}(s) &= \frac{m_c \langle \bar{s}g\sigma.Gs \rangle}{2^6 3 \pi^4 s} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\alpha+\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s], \\ \rho_2^{mix}(s) &= -\frac{m_s \langle \bar{s}g\sigma.Gs \rangle}{2^6 3 \pi^4 s} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \left[ 10m_c^2 - 2\alpha(1-\alpha)s - \frac{m_c^2 - \alpha(1-\alpha)s}{1-\alpha} - 5m_c^2 \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} \right], \\ \rho^{\langle \bar{s}s \rangle^2}(s) &= -\frac{\langle \bar{s}s \rangle^2}{36\pi^2} \left( \frac{5m_c^2}{s} - \frac{1}{2} \right) \sqrt{1-4m_c^2/s}, \end{aligned} \quad (12)$$

$$\begin{aligned} \rho^{mix\langle \bar{s}s \rangle^2}(s) &= -\frac{\langle \bar{s}s \rangle \langle \bar{s}g\sigma.Gs \rangle}{3^2 2^4 \pi^2 s} (1+4m_c^2/s) \sqrt{1-4m_c^2/s}, \\ \Pi^{mix\langle \bar{s}s \rangle}(M^2) &= -\frac{\langle \bar{s}s \rangle \langle \bar{s}g\sigma.Gs \rangle}{3^2 2^4 \pi^2} \left( \frac{2}{3} - 3 \int_0^1 d\alpha \exp \left[ -\frac{m_c^2}{\alpha(1-\alpha)M^2} \right] \left[ \alpha - 2\alpha^2 + \frac{2m_c^2}{M^2} \right] \right). \end{aligned} \quad (13)$$

where the integration limits are given by  $\alpha_{min} = (1 - \sqrt{1-4m_c^2/s})/2$ ,  $\alpha_{max} = (1 + \sqrt{1-4m_c^2/s})/2$  and  $\beta_{min} = \alpha m_c^2/(s\alpha - m_c^2)$ . The contribution of dimension-six condensates  $\langle g^3 G^3 \rangle$  is neglected, since it is suppressed by the loop factor  $1/16\pi^2$ . In Eq. (13) we have included, for completeness a part of the dimension-8 condensate, related with the mixed condensate-quark condensate contribution. We should note that a complete evaluation of these contributions require more involved analysis including a non-trivial choice of the factorization assumption basis [21]. In Eq. (13) the  $\Pi^{mix\langle \bar{s}s \rangle}(M^2)$  term is treated separately because its imaginary part is proportional to delta functions and, therefore, can be easily integrated [22].

It is very interesting to notice that the current in Eq. (1) does not get contribution from the quark condensates when  $m_s = 0$ . This is very different from the OPE behavior obtained to the scalar-diquark axial-antidiquark current used for the  $X(3872)$  meson in ref. [18], but very similar to the OPE behavior obtained for the axial double-charmed meson  $T_{cc}$ , also described by a scalar-diquark axial-antidiquark current [23].

### III. SUM RULE PREDICTIONS FOR $m_Y$

The values used for the quark masses and condensates are [17, 24]:  $m_c(m_c) = (1.23 \pm 0.05)$  GeV,  $m_s = (0.13 \pm 0.03)$  GeV,  $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3$  GeV<sup>3</sup>,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ ,  $\langle \bar{q}g\sigma.Gq \rangle = m_0^2 \langle \bar{q}q \rangle$  with  $m_0^2 = 0.8$  GeV<sup>2</sup>,  $\langle g^2 G^2 \rangle = 0.88$  GeV<sup>4</sup>.

We evaluate the sum rules in the Borel range  $2.8 \leq M^2 \leq 4.6$  GeV<sup>2</sup>, and in the  $s_0$  range  $5.0 \leq \sqrt{s_0} \leq 5.2$  GeV. To fix the continuum threshold range we extract the mass from the sum rule, for a given  $s_0$ , and accept such value of  $s_0$  if the obtained mass is around 0.5 GeV smaller than  $\sqrt{s_0}$ .

From Fig. 1 we see that we obtain a quite good OPE convergence for  $M^2 \geq 3.2$  GeV<sup>2</sup>. Therefore, we fix the lower value of  $M^2$  in the sum rule window as  $M_{min}^2 = 3.2$  GeV<sup>2</sup>. This figure also shows that

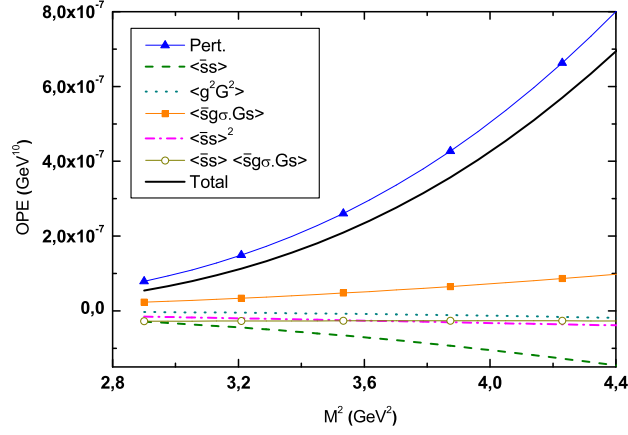


FIG. 1: The OPE convergence in the region  $2.8 \leq M^2 \leq 4.5 \text{ GeV}^2$  for  $\sqrt{s_0} = 5.1 \text{ GeV}$ . Perturbative contribution (solid line with triangles),  $\langle \bar{s}s \rangle$  contribution (dashed-line),  $\langle g^2 G^2 \rangle$  contribution (dotted line),  $\langle \bar{s}g\sigma.Gs \rangle$  contribution (solid line with squares),  $\langle \bar{s}s \rangle^2$  contribution (dot-dashed line),  $\langle \bar{s}g\sigma.Gs \rangle \langle \bar{s}s \rangle$  contribution (solid line with spheres) and total contribution (solid line).

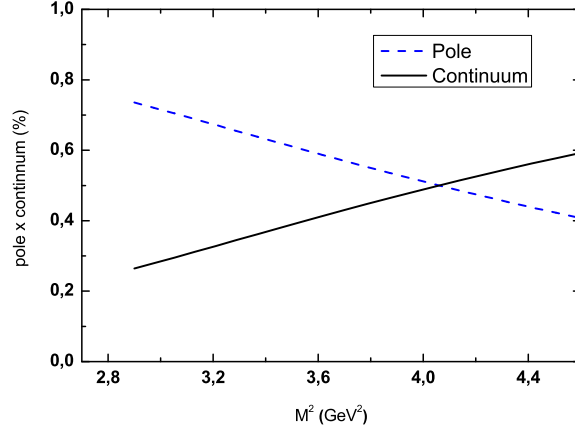


FIG. 2: The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for  $\sqrt{s_0} = 5.1 \text{ GeV}$ .

dimension-eight condensate contributions is very small. Since this is not a complete evaluation of the dimension-eight condensates contribution, it will be neglected in this calculation.

In Fig. 2 we show the comparison between pole and continuum contributions for  $\sqrt{s_0} = 5.1 \text{ GeV}$ , and we see that for  $M^2 \leq 4.05 \text{ GeV}^2$ , the pole contribution is bigger than the continuum contribution. Therefore, we fix  $M^2 = 4.05 \text{ GeV}^2$  as the upper limit of the Borel window for  $\sqrt{s_0} = 5.1 \text{ GeV}$ . The same analysis for the other values of the continuum threshold gives  $M^2 \leq 3.8 \text{ GeV}^2$  for  $\sqrt{s_0} = 5.0 \text{ GeV}$  and  $M^2 \leq 4.2 \text{ GeV}^2$  for  $\sqrt{s_0} = 5.2 \text{ GeV}$ . We then consider, for each value of  $s_0$ , the range of  $M^2$  values from  $3.2 \text{ GeV}^2$  until the one allowed by the pole dominance criteria given above.

To extract the mass  $m_Y$  we take the derivative of Eq. (10) with respect to  $1/M^2$ , and divide the result by Eq. (10). In Fig. 3, we show the  $Y$  meson mass, for different values of  $\sqrt{s_0}$ , in the relevant sum rule window, with the upper and lower validity limits indicated. From this figure we see that we get a very good Borel stability for  $m_Y$ .

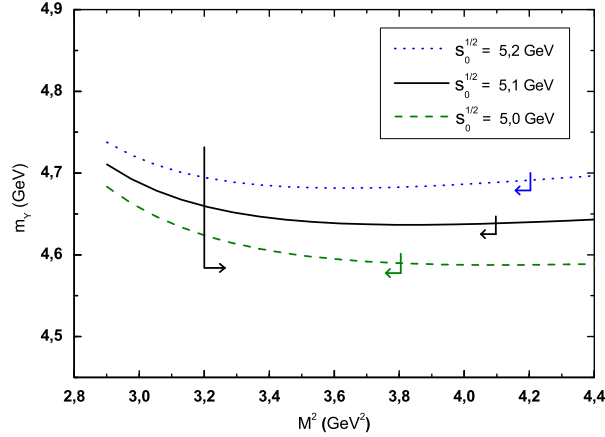


FIG. 3: The  $Y$  meson mass as a function of the sum rule parameter ( $M^2$ ) for different values of  $\sqrt{s_0}$ . The arrows indicate the region allowed for the sum rules: the lower limit (cut below  $3.2 \text{ GeV}^2$ ) is given by OPE convergence requirement and the upper limit by the dominance of the QCD pole contribution.

To check the dependence of our results with the value of the charm quark mass, we fix  $\sqrt{s_0} = 5.1 \text{ GeV}$  and vary the quark masses and the quark condensate in the range given above. We see that the results are not very sensitive to the changes in the values of the quark masses and condensate, the main source of uncertainty being the continuum threshold. Adding the errors in quadrature we finally get

$$m_Y = (4.65 \pm 0.10) \text{ GeV}, \quad (14)$$

in excellent agreement with the mass of the  $Y(4660)$  meson. Therefore we conclude that the meson  $Y(4660)$  can be described by a tetraquark state with a spin configuration given by scalar and vector diquarks.

#### IV. SUM RULE STUDY FOR $Y(4350)$

As pointed out in the introduction, from the di-pion invariant mass spectra shown in ref. [9], there is some indication that only the  $Y(4660)$  has a well defined intermediate state consistent with  $f_0(980)$ . The di-pion invariant mass spectra for the pions in the decay  $Y(4350) \rightarrow \psi' \pi^+ \pi^-$  could be consistent with a broad intermediate state with a mass around 600 MeV. This could be a  $\sigma$  scalar meson [25] and, in this case, one would not expect strange quarks in the current in Eq. (1).

Replacing the strange quarks in Eq.(1) by the generic light quark  $q$  and using  $m_q = 7.0 \text{ MeV}$ , we show, in Fig 4, the OPE convergence using  $\sqrt{s_0} = 4.9 \text{ GeV}$

Since now the quark condensate contribution is multiplied by the light quark mass, its contribution is now negligible. From this figure we see that we get a good OPE convergence for  $M^2 \geq 3.2 \text{ GeV}^2$ .

Again the upper limits for  $M^2$  are obtained by imposing that the pole contribution should be bigger than the continuum contribution. These values are given in Table I for each value of  $\sqrt{s_0}$ .

**Table I:** Upper limits in the Borel window for the sum rule with  $m_s = m_q$ .

$\sqrt{s_0} \text{ (GeV)}$	$M_{max}^2 \text{ (GeV}^2\text{)}$
4.8	3.5
4.9	3.8
5.0	4.0

In Fig. 5 we show the relative continuum (solid line) versus pole (dashed line) contribution, using  $\sqrt{s_0} = 4.9 \text{ GeV}$ , from where we clearly see that the pole contribution is bigger than the continuum contribution for  $M^2 < 3.5 \text{ GeV}^2$ .

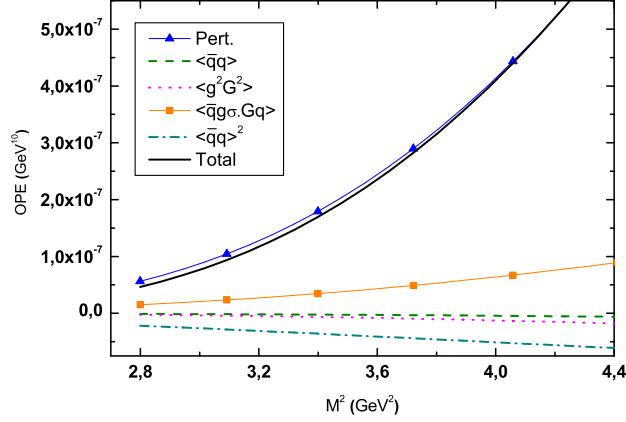


FIG. 4: The OPE convergence for the sum rule with  $m_s = m_q$ , using  $\sqrt{s_0} = 4.9$  GeV. The solid with triangles, dashed, dotted, solid with squares, dot-dashed and solid lines give, respectively, the perturbative, quark condensate, gluon condensate, mixed condensate, four-quark condensate and total contributions.

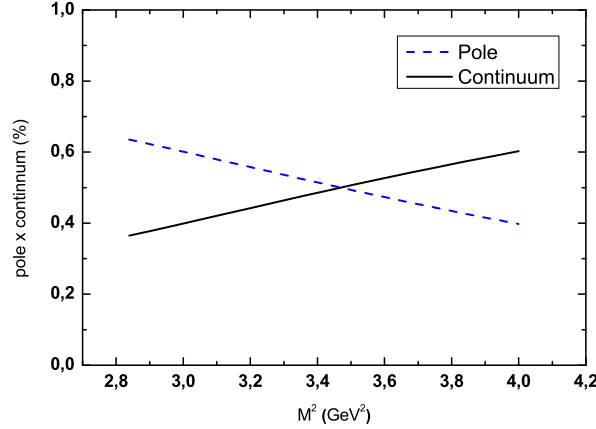


FIG. 5: Same as Fig. 2 for the sum rule with  $m_s = m_q$  and  $\sqrt{s_0} = 4.9$  GeV.

In this case the stability for the  $m_Y$  mass is not as good as in the previous case, but it is still acceptable, in the allowed sum rule window, as a function of  $M^2$  as can be seen by Fig. 6.

Taking into account the variations on  $M^2$ ,  $s_0$ ,  $\langle\bar{q}q\rangle$  and  $m_c$  in the regions indicated above we get:

$$m_Y = (4.49 \pm 0.11) \text{ GeV} , \quad (15)$$

which is bigger than the  $Y(4350)$  mass, but it is consistent with it considering the uncertainty.

## V. $D_{s0}\bar{D}_s^*$ MOLECULE

As pointed out in ref. [26], if the  $X(3872)$ , and  $Z^+(4430)$  are really molecular states, then many other molecules should exist. In particular a  $D_{s0}(2317)\bar{D}_s^*(2110)$  molecule with  $J^{PC} = 1^{--}$ , could also decay into  $\psi'\pi^+\pi^-$  with a dipion mass spectra consistent with  $f_0(980)$ . Therefore, in this section we consider a meson-meson current to check if this current could also describe the meson  $Y(4660)$ . A current with

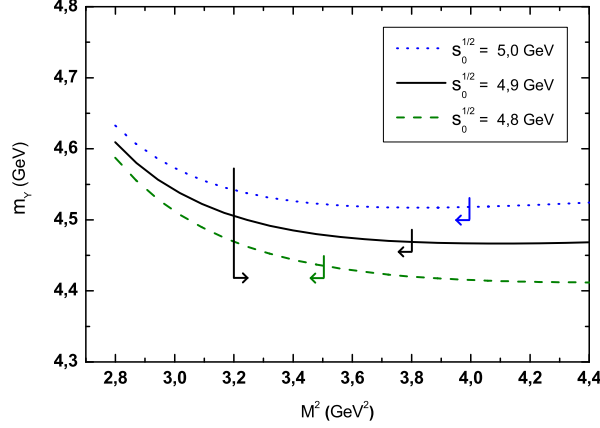


FIG. 6: The light  $m_Y$  meson mass as a function of the sum rule parameter ( $M^2$ ) for different values of  $\sqrt{s_0}$ . The arrows delimit the regions allowed for the sum rule.

$J^{PC} = 1^{--}$  and a symmetrical combination between the scalar and vector mesons is given by:

$$j_\mu = \frac{1}{\sqrt{2}} [(\bar{s}_a \gamma_\mu c_a)(\bar{c}_b s_b) + (\bar{c}_a \gamma_\mu s_a)(\bar{s}_b c_b)] . \quad (16)$$

With this current we get for the spectral density, results very similar to the ones in Eq.(12), the only differences being the color factors. We get a factor 3/4 for the diagrams where there is no gluons being exchanged between the quark lines, and a factor 3/2 for the diagrams with gluons being exchanged between the quark lines:

$$\begin{aligned} \rho^{pert}(s) &= -\frac{1}{2^{10}\pi^6 s} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s]^3 [m_c^2 - 2m_c^2(\alpha+\beta) + \alpha\beta s] , \\ \rho^{\langle \bar{s}s \rangle}(s) &= -\frac{9m_s m_c^2 \langle \bar{s}s \rangle}{2^6 \pi^4 s} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} [(\alpha+\beta)m_c^2 - \alpha\beta s] , \\ \rho^{\langle G^2 \rangle}(s) &= \frac{m_c^2 \langle g^2 G^2 \rangle}{2^{11} 3 \pi^6 s} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) \left[ 4(2\alpha+2\beta-1)m_c^2 - \frac{3m_c^2\beta}{\alpha}(1+\alpha+\beta) - \beta s(7\alpha-3\beta-3) \right] , \\ \rho_1^{mix}(s) &= \frac{m_c \langle \bar{s}g\sigma.Gs \rangle}{2^7 \pi^4 s} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\alpha+\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s] , \\ \rho_2^{mix}(s) &= -\frac{m_s \langle \bar{s}g\sigma.Gs \rangle}{2^7 \pi^4 s} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \left[ 5m_c^2 + \alpha^2 s - \frac{m_c^2}{1-\alpha} - 5m_c^2 \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} \right] , \\ \rho^{\langle \bar{s}s \rangle^2}(s) &= -\frac{\langle \bar{s}s \rangle^2}{2^4 3 \pi^2} \left( \frac{5m_c^2}{s} - \frac{1}{2} \right) \sqrt{1 - 4m_c^2/s} . \end{aligned} \quad (17)$$

With these changes, the mixed condensate is now, for the current in Eq. (16), more important than it was for the current Eq. (1), as can be seen in Fig. 7. For this current we find that the appropriate continuum threshold range is  $4.8 \leq \sqrt{s_0} \leq 5.0$  GeV and that the OPE convergence is very good for  $M^2 \geq 3.2$  GeV<sup>2</sup>. The upper limits for  $M^2$  are given in Table II for each value of  $\sqrt{s_0}$ .

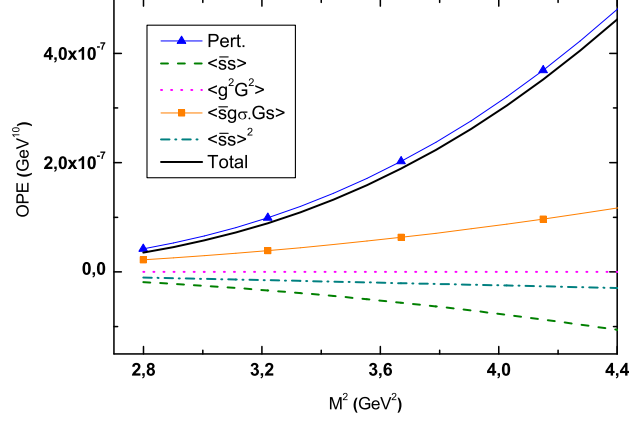


FIG. 7: The OPE convergence for the sum rule for the  $D_{s0}\bar{D}_s^*$  molecule with  $\sqrt{s_0} = 4.9$  GeV.

**Table II:** Upper limits in the Borel window for the sum rule for the  $D_{s0}\bar{D}_s^*$  molecule.

$\sqrt{s_0}$ (GeV)	$M_{max}^2$ (GeV <sup>2</sup> )
4.8	3.5
4.9	3.7
5.0	4.0

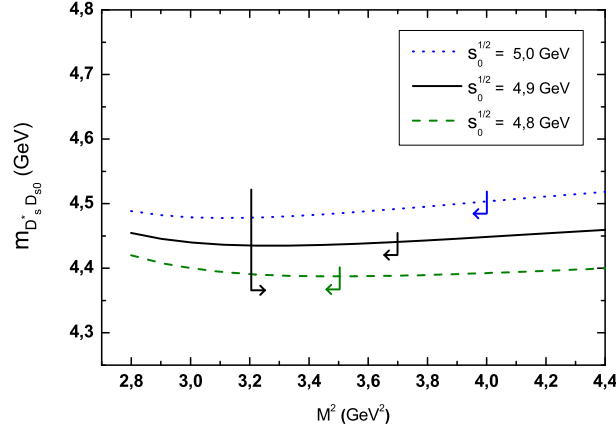


FIG. 8: The  $D_{s0}\bar{D}_s^*$  molecule mass as a function of the sum rule parameter ( $M^2$ ) for different values of  $\sqrt{s_0}$ . The arrows delimit the regions allowed for the sum rule.

From Fig. 8 we see that the Borel stability, as a function of  $M^2$  is very good, in the allowed sum rule window.

Taking into account the variations on  $M^2$ ,  $s_0$ ,  $\langle\bar{q}q\rangle$ ,  $m_s$  and  $m_c$  in the regions indicated above we get:

$$m_{D_{s0}\bar{D}_s^*} = (4.42 \pm 0.10) \text{ GeV} , \quad (18)$$

which is more in agreement with the  $Y(4350)$  mass than with the  $Y(4660)$  mass. It is important to mention that even if we use  $\sqrt{s_0} = 5.1$  GeV, which is the central value used for  $Y(4660)$ , we get  $m_{D_{s0}\bar{D}_s^*} \sim 4.5$  GeV, still in agreement, considering the errors, with the value in Eq. (18). Therefore,



we have to conclude that the  $Y(4660)$  meson is better explained with the diquark-antidiquark current in Eq. (1) than with the molecular current in Eq. (16). To conclude if we can associate this molecular state with the meson  $Y(4350)$  we need a better understanding of the di-pion invariant mass spectra for the pions in the decay  $Y(4350) \rightarrow \psi' \pi^+ \pi^-$ . From the spectra given in ref. [9], it seems to us that the  $Y(4350)$  is more consistent with a non-strange four-quark state than with a  $D_{s0} \bar{D}_s^*$  molecular state. However, from the mass obtained from the sum rules we see that we can explain the  $Y(4350)$  meson better as a  $D_{s0} \bar{D}_s^*$  molecular state.

It is also important to notice that the central mass obtained for the  $D_{s0} \bar{D}_s^*$  molecule is very close to the  $D_{s0}(2317) \bar{D}_s^*(2110)$  threshold, indicating that this channel might be forbidden in the  $Y(4350)$  decay.

## VI. $D_0 D^*$ MOLECULE

Similarly to what was done in section IV, we can consider a scalar-vector charmed mesons molecule:  $D_0 \bar{D}^*$  with  $J^{PC} = 1^{--}$ . For this we have only to change the strange quarks in Eq.(16) by the generic light quark  $q$ . In this case we get a good OPE convergence for  $M^2 \geq 3.2 \text{ GeV}^2$  and the continuum threshold in the range  $4.6 \leq \sqrt{s_0} \leq 4.8 \text{ GeV}$ . The upper limits for  $M^2$  are given in Table III for each value of  $\sqrt{s_0}$ .

**Table III:** Upper limits in the Borel window for the sum rule for the  $D_0 \bar{D}^*$  molecule.

$\sqrt{s_0} \text{ (GeV)}$	$M_{max}^2 \text{ (GeV}^2\text{)}$
4.6	3.3
4.7	3.5
4.8	3.7

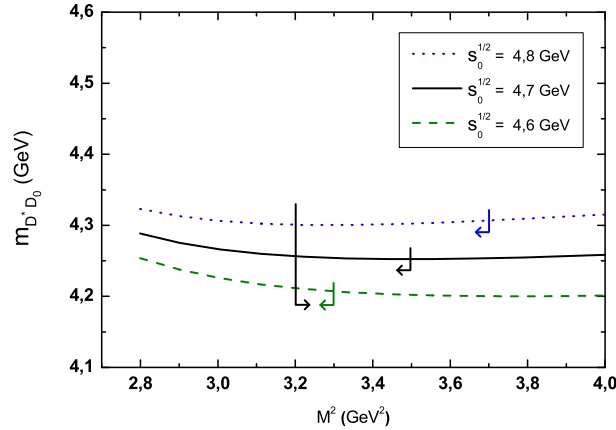


FIG. 9: The  $D_0 \bar{D}^*$  molecule mass as a function of the sum rule parameter ( $M^2$ ) for different values of  $\sqrt{s_0}$ . The arrows delimit the regions allowed for the sum rule.

Again we get a very good Borel stability as a function of  $M^2$ , in the allowed sum rule window, as can be seen by Fig. 9. Considering the variations on  $M^2$ ,  $s_0$ ,  $\langle \bar{q}q \rangle$  and  $m_c$  in the regions indicated above we get:

$$m_{D_0 \bar{D}^*} = (4.27 \pm 0.10) \text{ GeV} , \quad (19)$$

in excellent agreement with the mass of the meson  $Y(4260)$ . Again, to conclude if we can associate this molecular state with the meson  $Y(4260)$  we need a better understanding of the di-pion invariant mass spectra for the pions in the decay  $Y(4260) \rightarrow J/\psi \pi^+ \pi^-$ . From the spectra given in ref. [9], it seems that the  $Y(4260)$  is consistent with a non-strange molecular state  $D_0 \bar{D}^*$ .

Regarding charmed non-strange scalar mesons, a broad enhancement ( $D_0$ ) mass distribution has been observed [27]. Its mass is now compiled as [28]  $m_{D_0} = 2352 \pm 50 \text{ MeV}$ . Therefore, the  $D_0 \bar{D}^*$  threshold is around 2360 MeV and is 100 MeV above the molecule mass in Eq. (19).

## VII. CONCLUSIONS

In conclusion, we have presented a QCDSR analysis of the two-point functions for tetraquark charmonium states with  $J^{PC} = 1^{--}$ , to study the charmonium  $Y$  mesons recently observed by BaBar and BELLE Collaborations. We have considered two kinds of currents: a diquark-antidiquark tetraquark state with a symmetric spin distribution  $[cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}$  (where the generic quark  $q$  can be either a light  $u$  or  $d$  quark, or a strange,  $s$ , quark), and a molecular state with a symmetrical combination between scalar and vector charmed mesons.

Our findings indicate that the  $Y(4660)$  meson can be very well described by a diquark-antidiquark  $(cs)(\bar{c}\bar{s})$  tetraquark state. This quark content is consistent with the di-pion invariant mass spectra shown in ref. [9], which shows that there is some indication that the  $Y(4660)$  has a well defined di-pion intermediate state consistent with  $f_0(980)$ .

In the case of the  $Y(4350)$  meson, its mass can be reproduced, considering the errors, if we assume that it is a diquark-antidiquark  $(cq)(\bar{c}\bar{q})$  tetraquark state, or a  $D_{s0}\bar{D}_s^*$  molecule. Since an indication from its quark content can be obtained from the di-pion invariant mass spectra in the decay  $Y(4350) \rightarrow \psi'\pi^+\pi^-$ , we need a better understanding of it before we can reach a definite conclusion about its structure.

We also found that the  $Y(4260)$  meson can be very well described by a  $D_0\bar{D}^*$  molecular state.

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